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MODEL:
 ! Computing efficiency of Decision Making Units (DMU), using
Directional Distance Function. Ref:
 Chambers, R., Y. Chung, and R. Fare (1996), "Benefit and Distance Functions",
 Journal of Economic Theory, vol. 70, pp. 407-419.
For each DMU, we are given:
    How much it used of each input,
    How much it delivered of each output.
All DMU's use the same types of inputs and
deliver the same types of outputs.
A DMU i is efficient or undominated if there is no positively weighted
combination of other DMU's that:
  uses no more of every input, and
  delivers at least as much of every output.
A "Beta" score is computed for each DMU i which is
  <= 0 if DMU i is efficient or undominated,
  > 0 if DMU i is inefficient or dominated.
! Keywords: Benchmarking, DDF, Directional Distance Function,
  Data envelopment ANALYSIS, DEA, Efficiency,
            LINGO, Pareto optimal;
SETS:
 DMU: LAMBDA;
 FACTOR;
 FACTIN( FACTOR);
 DXF( DMU, FACTOR):  F ! F( I, J) = Jth factor of DMU I;
          ;
ENDSETS
DATA:
! School data;
  DMU = BL HW NT OP YK EL;  ! The schools;
! Inputs are spending/pupil, % not low income;
! Outputs are Writing score and Science score;
 FACTOR= COST RICH       WRIT  SCIN;
 FACTIN = COST RICH; ! Input factors;
!     The inputs,     the outputs;
  F  =  89.39  64.3     25.2   223  ! BL;
        86.25  99       28.2   287  ! HW;
       108.13  99.6     29.4   317  ! NT;
       106.38  96       26.4   291  ! OP;
        62.40  96.2     27.2   295  ! YK;
        47.19  79.9     25.5   222; ! EL;
 LOSCALE = 0;     ! Any bounds on SCALE;
 HISCALE = 9999;
ENDDATA

SUBMODEL DDF:
  @FREE( BETA);
  MAX = BETA;
! If there is no BETA >= 0 that satisfies both the input and output
set of constraints, it means that DMU ICHK is undominated. We could
not find any positively weighted combination of the other DMU's
that produced more of each output, and used less of each input;
! For the output measures;
  @FOR( FACTOR(j) | #NOT# @IN( FACTIN, j):
    @SUM( DMU( i) | i #NE# ICHK: LAMBDA(i)*F(i,j)) >= (1+BETA)*F(ICHK,j)
      );
! For the input measures;
  @FOR( FACTIN(j):
    @SUM( DMU( i) | i #NE# ICHK: LAMBDA(i)*F(i,j)) <= (1-BETA)*F(ICHK,j)
      );
 @SUM( DMU(i) | i #NE# ICHK: LAMBDA(i)) = SCALE;
! Some formulations require SCALE = 1 ...;
 @BND( LOSCALE, SCALE, HISCALE);
! but this can be misleading if there are two DMU identical
  except for scale;
ENDSUBMODEL

CALC:
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        @SET( 'TERSEO', 2);   !Minimal output;
        !Write out a solution report;
        @WRITE(@NEWLINE( 2),' A Decision Making Unit is Undominated if its Efficiency > 1.'
            ,@NEWLINE(1));
        @WRITE( '         DMU      Beta   Efficiency   Scale', @NEWLINE(1));

 ! Solve the DEA model for each DMU D;
    @FOR( DMU( D):
        ICHK = D;
        @SOLVE( DDF);   ! Solve the model above;
        @WRITE( ' ', @FORMAT( DMU( D),'10s'), '   ', @FORMAT( BETA, '6.3f'),
            '    ',@FORMAT(1-BETA,'9.5f'),'  ', @FORMAT(SCALE,'6.4f'), @NEWLINE(1));
        );
ENDCALC
END
! Any statements after the END statement are disregarded.
  To use any data set below, paste it into the DATA section above,
  replacing the previous data set;

! School data;
  DMU = BL HW NT OP YK EL;  ! The schools;
! Inputs are spending/pupil, % not low income;
! Outputs are Writing score and Science score;
 FACTOR= COST RICH      WRIT  SCIN;
 FACTIN = COST RICH; ! Input factors;
!     The inputs,    the outputs;
  F  =  89.39  64.3     25.2   223  ! BL;
        86.25  99       28.2   287  ! HW;
       108.13  99.6     29.4   317  ! NT;
       106.38  96       26.4   291  ! OP;
        62.40  96.2     27.2   295  ! YK;
        47.19  79.9     25.5   222; ! EL;
 LOSCALE = 0;    ! Any bounds on SCALE;
 HISCALE = 9999;


! Korean electrical utilities.  Taken from:
   Ray, S. (2012), "Data Envelopment Analysis:
   Theory and Techniques for Economics and Operations Research",
   Cambridge University Press;
 DMU = U01 U02 U06 U07 U08 U11 U12 U25 U30; ! Decision Making Units;
 FACTOR= CAPITAL  LABOR    FUEL   OUTPUT;
    F = 706.698 643.389 648.946 614.661 ! U01;
        1284.9  1142.2 1101.65 1128.39 ! U02;
        1027.92  527.72  448.10  404.99 ! U06;
        2055.85 1048.22 2136.09 2276.89 ! U07;
        2055.85 1055.45 2140.03 2278.26 ! U08;
         51.396 101.207  91.632  73.405 ! U11;
        51.3962 93.9782 91.9232 73.8759 ! U12;
        906.033 780.742 554.623 673.120 ! U25;
        2569.81 1763.90 3352.76  3528.04;!U30;
 FACTIN = CAPITAL LABOR FUEL;  ! Input factors;
 LOSCALE = 0;    ! Any bounds on SCALE;
 HISCALE = 99999;

! Airline data;
!  Taken from: Coelli, T., E. Griffel Tatje, and S. Perelman (2002),
 "Capacity Utilization and Profitability: A Decomposition of Short-run Profit
Efficiencey",
 International Journal of Production Economics, vol. 79, pp. 261-278;
! Decision Making Units;
 DMU = NIPPON CATHAY JAL QANTAS SAUDIA SINGAPORE AUSTRIA BRITISH FINNAIR LUFTHANSA
AMERICAN DELTA UNITED;
 FACTOR= PASS CARGO  LAB   FUEL   MATL     CAP;
 FACTIN= LAB FUEL MATL CAP; ! Input factors;
  F=   35261    614  12222    860  2008    6074 ! NIPPON;
       23388   1580  12214    456  1492    4174 ! CATHAY;
       57290   3781  21430   1351  2536   17932 ! JAL;
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      28991   1330   17997    393   1474    4784 ! QANTAS;
      18969    760   24708    235    806    6819 ! SAUDIA;
      32404   1902   10864    523   1512    4479 ! SINGAPORT;
       2943     65    4067     62    241     587 ! AUSTRIA;
      67364   2618   51802   1294   4276   12161 ! BRITISH;
       9925    157    8630    185    303    1482 ! FINNAIR;
      50989   5346   45514   1078   3314    9004 ! LUFTHANSA;
     133796   1838   80627   2381   5149   18624 ! AMERICAN;
      96540   1300   61675   1997   3972   14063 ! DELTA;
     131905   2326   73902   2246   5678   18204;! UNITED;
 LOSCALE = 0;     ! Any bounds on SCALE;
 HISCALE = 9999;

   ! Two DMU equivalent except for scale;
   DMU = DMLIL DMUBIG;
   FACTOR = INPUT1 OUTPUT1;
   FACTIN = INPUT1;
   F =         1       1 ! DMULIL;
               9       9;! DMU2BIG;
 LOSCALE = 1;     ! Any bounds on SCALE;
 HISCALE = 1;
! LOSCALE = 0;     ! Any bounds on SCALE;
! HISCALE = 99999;
! If we restrict SCALE = 1, then DMULIL is declared substantially more efficient;
! If we do not restrict SCALE, then DMULIL and DMUBIG are
 declared equally efficient;

! Other DEA references;

! Benneyan, J., M. Ceyhan, and A. Sunnetci (2007), "Data Envelopment Analysis of
National Healthcare Systems and their Relative Efficiencies", Proceedings of the
37th International Conference on Computers and Industrial Engineering, Alexandria,
Egypt, edited by M. Elwany and A. Eltawil.;

! Beriha, G., B. Patnaik, and S. Mahapatra (2011), "A Data Envelopment Analysis
 Approach for Benchmarking of Safety Performance",
2011 3rd Int. Conference on Information and Financial Engineering, Singapore;

! El-Mashaleh, M. S. Rababeh, and K. Hyari,(2010),"Utilizing data envelopment
analysis to benchmark safety performance of construction contractors",
 International Journal of Project Management, vol 28, pp. 61-67.;

! Hermans, E., T. Brijs, G. Wets, and K. Vanhoof, (2008)
 "Benchmarking Road Safety: Lessons to Learn from a Data Envelopment Analysis",
 Hasselt University, Belgium.;
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